Probabilistic Discovery of Articulated Object Kinematics Using Trajectory Matching with a pseudo-Riemannian Metric on $SE(3)$

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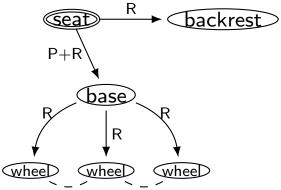
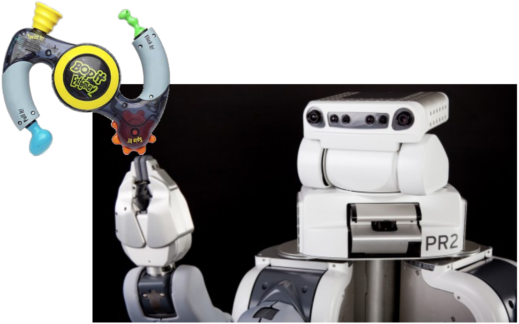
Motivation

OR: a map of the rabbit hole

- End goal: robot interacts with real world object and learns a kinematic tree
- Input: feature trajectories $x_i(t) \in SE(3)$
- Output: kinematic tree $Rigid(1, Prismatic(2, Revolute(3, 4)))$
- Key subproblem: fit several candidate joint models to a set of feature trajectories, and decide on the best model
- Sticking point 1: how do we compare the observed and predicted trajectories of a feature? We need to be able to compare elements of $SE(3)$.
- Sticking point 2: how do we determine which sub-objects are connected?





Literature Review

- Interactive Perception (Katz et al 2008, 2012)
  - Perception and action are not as decoupled as roboticists like to pretend
  - Tracking: optical flow, Lucas-Kanade registration, SIFT features
  - Segmentation: weighted max-flow/min-cut (?)
  - Fitting: ad-hoc rigid/prismatic/revolute
- Motion subspaces (Yan & Pollefeys 2006)
  - Joints restrict the motion of object parts to intersecting subspaces of $SE(3)$
  - Tracking/segmentation: bypassed (input is trajectories)
  - Fitting: estimate subspace of each feature, build graph using the principle angles between all subspaces, then minimum spanning tree
- Probabilistic approach (Sturm et al 2011)
  - Bayesian treatment of the trajectory matching problem
  - Main inspiration for the current paper
  - Tracking/segmentation: augmented reality markers
  - Fitting: nonlinear optimization using kinematics, then minimum spanning tree on BIC

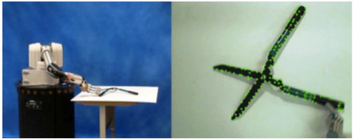
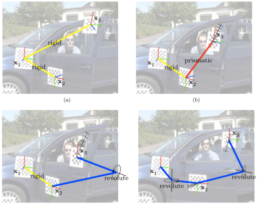


Fig. 1. The mobile manipulator UMan interacts with a tool, extracting the tool's kinematic model to enable purposeful manipulation. The right image shows the scene as seen by the robot through an overhead camera; dots mark tracked visual features.



Probabilistic Joint Fitting

- Input is trajectories $X = \{\bar{x}_t^k \in SE(3) \mid k \in \{1..K\}, t \in \{1..T\}\}$
- Ouptut is graph $G = (V, E)$ where $V \in \{1..K\}$ and $E = \{M = (J, \theta, \sigma)^i \mid i \in \{1..N\}\}$
- Now, the math:

$$
\begin{aligned}
\widehat{E} &= \max_{E} P(X \mid E) P(M) \\
&= \max_{E \in S} \prod_{i=1}^{K-1} \max_{M^i} P(X \mid M^i) P(M^i) \\
&= \max_{E \in S} \prod_{i=1}^{K-1} \max_{M^i} \prod_{t=1}^{T} P(\bar{\Delta}_t^{a^i:b^i} \mid M^i) P(M^i) \\
&= \max_{E \in S} \sum_{i=1}^{K-1} \max_{M^i} \sum_{t=1}^{T} \log P(\bar{\Delta}_t^{a^i:b^i} \mid M^i) + \log P(M^i) \\
&\approx \min_{E \in S} \sum_{i=1}^{K-1} \min_{M^i} \sum_{t=1}^{T} ||\bar{\Delta}_t^{a^i:b^i} - fk_{J^i}(\theta^i, \sigma_t^i)|| + |\theta^i|
\end{aligned}
$$

Distance Metric

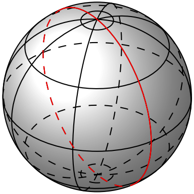
- For minimization, we need to answer this question: given $\bar{a}_{1..T}, \bar{b}_{1..T}$ two trajectories in $SE(3)$, what is the "distance"?

$$||\bar{a}_{1..T} - \bar{b}_{1..T}|| = \sum_{t=1}^{T} ||\bar{a}_t - \bar{b}_t||$$

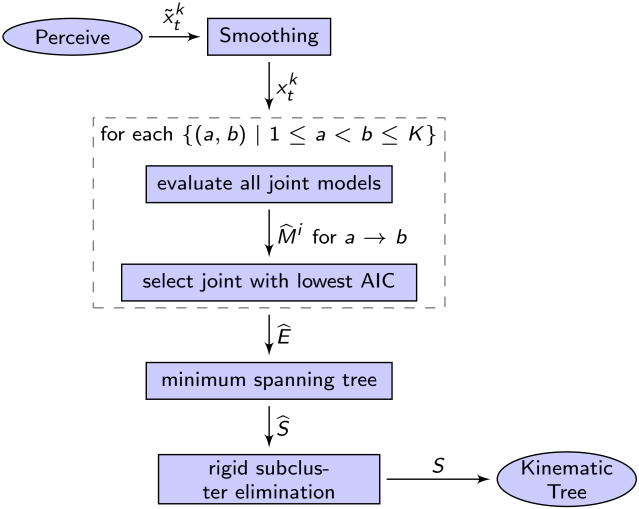
- Can we just subtract the parameters? $\sqrt{(a_x - b_x)^2 + (a_y - b_y)^2 + (a_z - b_z)^2 + (a_\theta - b_\theta)^2 + (a_\phi - b_\phi)^2 + (a_\alpha - b_\alpha)^2}$
- No good! The units are incompatible, plus subtracting angles is a leading cause of dinosaur attacks.
- Solution: since $SE(3)$ is a Lie group, evaluate $||x - y||$ as a "line integral" of distances computed along a path in the Lie algebra $\mathfrak{se}(3)$.
- The formula, from Park 1995, is

$$||\bar{a} - \bar{b}|| = \sqrt{c||\log(A_R^T B_R)||_F^2 + d||\vec{a}_T - \vec{b}_T||^2}$$

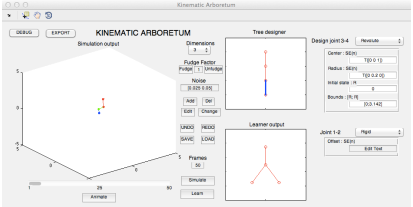
(We still have to make up an arbitrary conversion factor $\frac{c}{d}$.)



Putting it all together

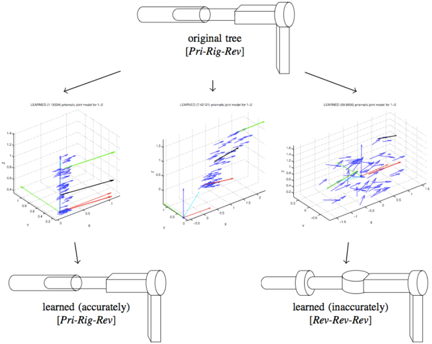


Experiment 1: Simulation



- Tree designer GUI used to debug and experiment with simulation
- Control over simulation parameters: T, inflation, noise

- Main experiment: sensitivity of learning to noise and inflation
- Figure shows the simulation of a prismatic joint at three noise levels and the corresponding learning output



Experiment 2: Real World

```
k shoulder -100 elbow -100 wrist 100 base -100
z 5
k base 75
z 5
k wrist 30
z 3
k wrist -60
z 3
k wrist 30
z 2
j
z 5
```