3D Force Diorama
Five Scenarios

1. Two ice-skaters are standing on the ice at the Spectrum after a Flyers win. Skater 1 has a weight of $1500N$ and Skater 2 other has a mass of $75kg$. After the game, the zamboni hasnt run yet so the ice has a coefficient of friction $\mu_k = .13$. Skater 1 pushes Skater 2 away with a force of $500N$, what happens to each skater? How much force does Skater 2 apply to Skater 1? What is the magnitude and direction of each skaters acceleration?

   (a) How much force does Skater 2 apply to Skater 1? What happens to each skater?
   Skater 1 exerts $500N$ of force on Skater 2 when she pushes Skater 1. Therefore, Skater 2 exerts $500N$ of force on Skater 1 but in the opposite direction. According to Newton's 3^{rd} Law, every action has an equal and opposite reaction. Therefore, Skater 1 exerts a force of $500N$ on Skater 2 and Skater 2 exerts a force of $-500N$ on Skater 1. The negative sign indicates direction. Each skater moves away from their starting location; Skater 1 moves in the negative direction and Skater 2 moves in the positive direction.

   (b) What is the magnitude and direction of each skater's acceleration?
   Given: $F_{g,E\to S_1} = 1500N$; $m_{S_2} = 75kg$; $F_{S_1\to S_2} = 500N$

For Skater 1...	For Skater 2...
$\Sigma F_y = 0 = F_{\perp,I\to S_1} + (-F_{g,E\to S_1})$	$\Sigma F_y = 0 = F_{\perp,I\to S_2} + (-F_{g,E\to S_2})$
$F_{\perp,I\to S_1} = F_{g,E\to S_1} = 1500N$	$F_{\perp,I\to S_2} = F_{g,E\to S_2} = (75kg)(9.8\frac{N}{kg}) = 735N$
$F_f = \mu F_\perp$	$F_f = \mu F_\perp$
$F_{f,I\to S_1} = .13F_{\perp,I\to S_1}$	$F_{f,I\to S_2} = .13F_{\perp,I\to S_2}$
$F_{f,I\to S_1} = 195N$	$F_{f,I\to S_2} = 95.55N$
$\Sigma F_x = m_{S1}a_x = -F_{a,S_2\to S_1} + F_{f,I\to S_1}$	$\Sigma F_x = m_{S2}a_x = F_{a,S_1\to S_2} - F_{f,I\to S_2}$
$\Sigma F_x = m_{S1}a_x = -500N + 195N$	$\Sigma F_x = m_{S2}a_x = 500N - 95.55N$
$\frac{1500N}{9.8\frac{N}{kg}}a_x = -305N$	$(75kg)a_x = 404.45N$
$a_x = -1.993\frac{m}{s^2}$	$a_x = 5.39\frac{m}{s^2}$

2. Two ice-skaters are playing tug-of-war in the Rockefeller Center ice-skating rink. Skater 1 has a weight of $245N$ and Skater 2 has a weight of $735N$. The smooth, new ice has a coefficient of friction of $\mu_k = .05$. Skater 2 pulls on the rope with a force of $300N$, what happens to each skater? How much force does the rope apply to Skater 2? What is the magnitude and direction of each skaters acceleration?

(a) What happens to each skater?
The skaters move toward the center of the rope.

(b) How much force does the rope apply to Skater 2?
The rope applies $-300N$ of force to Skater 2. It applies an equal amount of force to Skater 2 as it does to Skater 1 except in the opposite direction. We know this because of Newton's 3^{rd} Law.

(c) What is the magnitude and direction of each skater's acceleration?
Given: $F_{g,E\to S_1} = 245N$; $F_{g,E\to S_2} = 735N$; $F_{S_2\to S_1} = 300N$

For Skater 1...	For Skater 2...
$\Sigma F_y = 0 = F_{\perp,I\to S_1} + (-F_{g,E\to S_1})$	$\Sigma F_y = 0 = F_{\perp,I\to S_2} + (-F_{g,E\to S_2})$
$F_{\perp,I\to S_1} = F_{g,E\to S_1} = 245N$	$F_{\perp,I\to S_2} = F_{g,E\to S_2} = 735N$
$F_f = \mu F_\perp$	$F_f = \mu F_\perp$
$F_{f,I\to S_1} = .05F_{\perp,I\to S_1}$	$F_{f,I\to S_2} = .05F_{\perp,I\to S_2}$
$F_{f,I\to S_1} = 12.25N$	$F_{f,I\to S_2} = 36.75N$
$\Sigma F_x = m_{S1}a_x = F_{a,S_2\to S_1} - F_{f,I\to S_1}$	$\Sigma F_x = m_{S2}a_x = -F_{a,S_1\to S_2} + F_{f,I\to S_2}$
$\Sigma F_x = m_{S1}a_x = 300N - 12.25N$	$\Sigma F_x = m_{S2}a_x = -300N + 36.75N$
$\frac{245N}{9.8\frac{N}{kg}}a_x = 287.75N$	$\frac{735N}{9.8\frac{N}{kg}}a_x = -263.25N$
$a_x = 11.51\frac{m}{s^2}$	$a_x = -3.51\frac{m}{s^2}$

3. A large cannon with a mass of $1650kg$ is waiting on the railroad tracks outside a castle to stop any passers by. The cannon accidentally misfires and shoots a $35kg$ cannon ball horizontally. The railroad tracks have a coefficient of friction $\mu_s = 1.4$. The cannon exerts a force of $8400N$ on the cannon ball when it is shot out. What force does the cannon ball exert on the cannon? What is the magnitude and direction of the cannon and the cannon balls horizontal acceleration?

(a) What force does the cannon ball exert on the cannon?
The cannon ball exerts the same amount of force on the cannon and the cannon does on the cannon ball but in the opposite direction due to Newton's 3^{rd}. The cannon ball exerts $-8400N$ of force on the cannon. The negative sign shows direction.

(b) What is the magnitude and direction of the cannon and the cannon ball's horizontal acceleration? Given: $m_c = 1650kg$; $m_b = 35kg$; $F_{c\to b} = 8400N$

For cannon...	For cannon ball...
$\Sigma F_y = 0 = F_{\perp,T\to c} + (-F_{g,E\to c})$	$\Sigma F_y = (-F_{g,E\to b}) = ma$
$F_{\perp,T\to c} = F_{g,E\to c} = ma$	$-F_{g,E\to b} = ma$
$= (1650kg)(9.8\frac{N}{kg}) = 16170N$	$F_{g,E\to b} = (35kg)(9.8\frac{N}{kg}) = 343N$
$F_f = \mu F_\perp$	$F_f = \mu F_\perp$
$F_{f,T\to c} = 1.4F_{\perp,T\to c}$	$F_f = 0N$
$F_{f,T\to c} = 22638N$	
$\Sigma F_x = m_c a_x = -F_{a,b\to c} + F_{f,T\to c}$	$\Sigma F_x = m_b a_x = F_{a,c\to b}$
$\Sigma F_x = m_c a_x = -8400N + 22683N$	$\Sigma F_x = m_b a_x = 8400N$
$(1650kg)a_x = 14283N$	$(35kg)a_x = 8400N$
$a_x = 8.66\frac{m}{s^2}$	$a_x = 240\frac{m}{s^2}$

The cannon does *NOT* actually move. The cannon ball does not apply enough force to overcome static friction.

4. Flo is vacationing in her rowboat on Lake Erie and enjoying the Cleveland skyline. Flo exercises regularly and only weighs $535N$. Her boat on the other hand isnt so fit and has a mass of $78kg$. Flo decides to cool off on this hot summer day and dives into the lake. The cold water has a coefficient of friction $\mu_k = .70$ between the boat and the water. As she jumps off, the boat exerts a force of $720N$ on her. What force does Flo exert on the boat? What is the magnitude and direction of Flos and the boats acceleration?

   (a) What force does Flo exert on the boat?
   Flo exerts a force of $-720N$ on the boat. It is an equal and opposite force to that of the force exerted by the boat on Flo (Newton's 3^{rd} Law).

   (b) What is the magnitude and direction of Flo's and the boat's acceleration?
   Given: $m_b = 78kg$; $F_{g,E\to F} = 535N$; $F_{b\to F} = 720N$

For the boat...	For Flo...
$\Sigma F_y = 0 = F_{\perp,w\to b} + (-F_{g,E\to b})$	$\Sigma F_y = (-F_{g,E\to F}) = ma$
$F_{\perp,w\to b} = F_{g,E\to b} = ma$	$-F_{g,E\to F} = ma$
$= (78kg)(9.8\frac{N}{kg}) = 764.4N$	$F_{g,E\to F} = 535N$
$F_f = \mu F_\perp$	$F_f = \mu F_\perp$
$F_{f,w\to b} = .70F_{\perp,T\to c}$	$F_f = 0N$
$F_{f,w\to b} = 535.08N$	
$\Sigma F_x = m_b a_x = -F_{a,F\to b} + F_{f,w\to b}$	$\Sigma F_x = m_F a_x = F_{a,b\to F}$
$\Sigma F_x = m_b a_x = -720N + 535.08N$	$\Sigma F_x = m_F a_x = 720N$
$(78kg)a_x = -184.92N$	$\frac{535N}{9.8\frac{N}{kg}}a_x = 720N$
$a_x = -2.37\frac{m}{s^2}$	$a_x = 13.19\frac{m}{s^2}$

5. Two men who reenact medieval battles in their free time are relaxing on separate boats ($35kg$ each). Suddenly, Lancelot grabs his jousting stick (that hes secretly packed) and challenges Count de Monet to a jousting competition. Unfortunately Count de Monet ($120kg$) did not bring his jousting stick. Lancelot and his jousting stick have a mass of $70kg$. Lancelot jabs Count de Monet in the chest with a force of $540N$. The water has a coefficient of friction $\mu_s = .70$. Without his stick, how much force does Count de Monet exert on the jousting stick? What is the magnitude and direction of the acceleration of the Count and the boat? Assume that both people do not fall out of either boat.

   (a) How much force does Count de Monet exert on the jousting stick? The Count exerts $-540N$ of force on the jousting stick. The amount of force the Count exerts on the stick is equal and opposite to that of the amount of force the stick exerts on the Count. We know this must true because of Newton's 3^{rd} Law.

   (b) What is the magnitude and direction of the acceleration of the Count and his boat?
   Given: $m_{L+b} = 105kg$; $m_{C+b} = 155kg$; $F_{L\to C} = 540N$

For Lancelot	For Count de Monet...
$\Sigma F_y = 0 = F_{\perp,w\to L+b} + (-F_{g,E\to L+b})$	$\Sigma F_y = 0 = F_{\perp,w\to C+b} + (-F_{g,E\to C+b})$
$F_{\perp,w\to L+b} = F_{g,E\to L+b} = ma$	$F_{\perp,w\to C+b} = F_{g,E\to C+b} = ma$
$= (105kg)(9.8\frac{N}{kg}) = 1029N$	$= (155kg)(9.8\frac{N}{kg}) = 1519N$
$F_f = \mu F_\perp$	$F_f = \mu F_\perp$
$F_{f,w\to L+b} = .70F_{\perp,w\to L+c}$	$F_{f,w\to C+b} = .70F_{\perp,w\to C+b}$
$F_{f,w\to L+c} = 720.3N$	$F_{f,w\to C+b} = 1063.3N$
$\Sigma F_x = -F_{a,C+b\to L+b} + F_{f,w\to L+b}$	$\Sigma F_x = F_{a,L+b\to C+m} - F_{f,w\to C+m}$
$m_{L+b}a_x = -540N + 720.3N$	$m_{C+b}a_x = 540N - 1063.3N$
$(105kg)a_x = 180.3N$	$(155kg)a_x = -523.3N$
$a_x = 1.72\frac{m}{s^2}$	$a_x = -3.38\frac{m}{s^2}$

   The Count does *NOT* actually move. Lancelot and his stick do not apply enough force to overcome static friction.