Net Present Value

EAY = $(1 + APR / n)^n - 1$; EAY = e^{APR} if interest is compounded continuously.

$FV_T = \sum C_t (1 + r)^{T-t}$

(N)PV = $\sum C_t / (1 + r)^t$

Perpetuity: PV = C / r; Growing Perpetuity: PV = $C_1 / (r - g)$

Annuity: PV = $C / r * (1 - 1 / (1+r)^T)$ [T is the number of payments received (starting next period)]

Growing Annuity: PV = $C / (r - g) * (1 - ((1 + g) / (1 + r))^T)$

Annuity in Advance: PV = $C_0 + C_1/r*(1 - 1/(1 + r)^{T-1}$

Infrequent Annuity: $r = (1+r_0)^n - 1$, $T = T_0/n$, where n is the number of periods between payments.

Inflation: (1 + r) = (1 + R)/(1 + i)

Capital Budgeting

Internal Rate of Return (IRR): $0 = \sum C_t / (1 + IRR)^t$.

Average Accounting Return: AAR = Average Profit / Average Investment

Profitability Index (PI): PI = $(\sum C_t / (1 + r)^t) / C_0$ (where the sum does not include C_0).

Cash Flows

Quick and Dirty: $C_t = EAT_t + Dep_t$

More Detailed: $C_t = R_t - CC_t - tax_t = R_t - CC_t - T_C(R_t - CC_t - Dep_t)$

$= (R_t - CC_t)(1 - T_C + T_C * Dep_t$ = EBIT + Tax Shield

Even more detailed:

Capital Gains on Machine = Sale Price – Book Value

Book Value = Price – Accumulated Depreciation

Cash from Sale of Machine = Sale Price – Capital Gains Tax

= Sale Price – Tax Rate * Capital Gains

Net Working Capital = Accounts Receivable – Accounts Payable + Inventory + Cash

Operating Costs = Fixed Costs + Variable Costs

Total Investment Cash Flow = Machine + Opportunity Cost + Change in NWC

Income Before Taxes = Sales – Costs – Depreciation

Cash Flow from Operations = Sales – Costs – Taxes

Bonds

Bonds: $P = \sum C / (1 + r_t)^t + F / (1 + r_T)^T$

Current Yield = Annual Payment / Price = Yield to Maturity – Appreciation to Maturity

YTM: $P = \sum C / (1 + YTM)^t + F / (1 + YTM)^T$

Spot rate: $r_n = (F / P_n)^{1/n} - 1$ (where P_n is the price of an n-year zero coupon bond).

Forward rate: ${}_{n-1}f_n = (1 + r_n)^n / (1 + r_{n-1})^{n-1} - 1$

Arbitrage: $\sum C_{ti} n_i = C_{t0}$ for each t.

Stocks

Stocks: Price = $\sum Div_t / (1 + r)^t$

$Return_t = Div_t / P + g$ = Dividend Yield + Capital Gain = $Div / P_0 + (P_1 - P_0)$

- Payout Ratio = Div / EPS; RR = PB = 1 – Payout Ratio = REPS / EPS
- g = RR * ROE; $EPS_{t+1} = EPS_t + RR * ROE$
- Price = EPS / r + NPVGO; P / E = 1 / r + NPVGO / EPS

Valuing Portfolios:

- $E(r) = \sum r_t / n$ or $E(r) = \sum P(r_i) r_i$
- $Var(r_i) = \sum (r_t - E(r))^2 / (n - 1)$ or $E(r) = \sum P(r_i) (r_i - E(r))^2$; $\sigma_i = \sqrt{Var(r_i)}$
- $Cov(r_A, r_B) = \sigma_{AB} = \sum P(r_A, r_B) (r_A - E(r_A))(r_B - E(r_B))$; $Corr(r_A, r_B) = \rho_{AB} = \sigma_{AB} / \sigma_A\sigma_B$
- $E(r_p) = \sum X_i E(r_i)$ where X_i is the percentage of security i in the portfolio.
- $Var(r_p) = \sum X_i X_j \sigma_{ij} = \sum X_i^2 \sigma_i^2 - \sum_{i \neq j} X_i X_j \sigma_{ij} = \beta_p \sigma_m^2 + \sigma_\varepsilon^2$
- Diversification Effect = $Var(r_p) - \sum X_i \sigma_i$

CAPM:

- Minimum Variance Portfolio (2): $X_A = (\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B) / (\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B)$
- Security Market Line (SML): $E(r_i) = r_f + \beta_i (E(r_m) - r_f)$
  - $\beta_i = \sigma_{im} / \sigma_m^2 = \rho_{im} \sigma_i / \sigma_m$; $\beta_p = \sum X_i \beta_i$
- Capital Market Line (CML): $E(r_p) = r_f + (E(r_m) - r_f) / \sigma_m * \sigma_p$
  - $E(r_p) = (1 - X) r_f + X E(r_S)$
  - $\beta_p\sigma_m$ is the market risk.

r_A = EBIT / Assets

- $\beta_{overall}$ = Assets / (Assets + Tax Shield) β_A + TS / (A + TS) β_{TS}
  - Assuming $\beta_B, \beta_{TS} \approx 0$, and debt is constant,
    - No taxes: $\beta_S = \beta_{Asset}$ (1 + B/S)
    - Taxes: $\beta_S = \beta_A + B/S (\beta_A - \beta_B)(1 - t_C) \approx \beta_A (1 + B(1-t_C)/S)$

Cost of Equity (r_E or r_S): Value of Stock (S) = # Shares * Market Price

- Under constant growth, $P_0 = Div_1 / (r_E - g)$, and $r_E = Div_1 / P_0 + g$
- For preferred stock, r_E = Div / P [constant dividends]
- Using CAPM: $r_E = r_f + \beta_E (r_m - r_f)$
- Or we may consider earnings: $r_S = (1 - t_C)(EBIT - r_B B) / S$
- $r_S = r_A + B/S (1 - t_C)(r_A - r_B) = V_U/E\ r_A - B/S (1 - t_C) r_B$

Weighted Average Cost of Capital:

- $r_{WACC} = S / (S + B)\ r_S + B / (S + B)\ r_B(1 - T_C)$
- r_{WACC} = Assets / V_L * $r_A + t_c B/V_L\ r_B = B/V_L\ r_B(1 - t_C) + S/V_L * r_S$
- $r_{WACC} = r_A$ if not taxes; $r_{WACC} = r_A(1 - t_C B/V_L)$ if taxes

Value of Company =

- PV(Assets) = PV(financial assets and claims)
  - $V_U = \sum C_t / (1 + r_A)^t$, where C_t is the unlevered cash flow.
- PV(tax shield) = $t_C r_B B/r_B = t_C B$
- Modigliani – Miller Propositions:

	Proposition I: Values	Proposition II: Returns
No Taxes	$V_U = V_L$	$r_A = r_{WACC} = S/(S+B)r_S + B/(S+B)r_B$
Taxes	$V_L = V_U + t_C B$	$r_S = r_A + B/S (r_A - r_B)(1 - t_C)$ $r_{WACC} = S/(S+B)r_S + B/(S+B)r_B(1 - t_C)$

- Miles-Ezzell: Assuming B/V is constant,
  - PV(tax shield) = $\sum t_c r_B B_{t-1} / (1+r_A)^{t-1}(1+r_B)$.
  - $r^* = r_A - t_C r_B (B/V)(1+r_A)/(1+r_B)$
  - V = NPV(Project) + Initial Investment

Valuation Methods:

- WACC: $V_L = \sum C_t / (1 + r^*)^t$, where C_t is the unlevered cash flow (no tax shield included).
  - If D/V is changing: Find r_S from r_B and r_A: $r_S = r_A + (B_t - PVTS_t)/S\ (r_A - r_B)$
- Adjusted Present Value:
  - APV = NPV(project) + NPV(Financing) = V_U + PV(financing) = $UCF(1 - t_C) / r_A - t_C B$
  - Levered Cash Flow = Net Income (Sales, costs, interest) – Tax (with depreciation and interest tax shields) – Investment (but not depreciation!); this is what goes to investors
    - Operating Income is before interest, net income is afterward.
  - Total Payout = Levered Cash Flow + Interest
  - If there is a target B/V ratio, then B = (target B/V) V_L. Since V_L depends on B (through tax shields), this requires solving for each in terms of the other.
- Flow to Equity: $V_L = LCF(1 - t_C) / r_S + B$ [assuming a perpetuity]
  - LCF = EBIT – interest
  - Cash Flow to Stockholders: $C_E = (X - r_B B)(1 - t_C)$ – debt principal.

Costs of Financial Distress

$V_L = V_M + V_N$ = value of marketable securities (stocks/bonds) + value on non-marketable (tax shield, costs of financial distress)

Dividend Policy

Retained Earnings + Cash from New Securities = Dividends + Investment

Lintner's Dividend Model: $D_t = a(p^*E_t) + (1 - a) D_{t-1}$

Options

Call Option: Payoff = max{0, S_T – X}; Put Option: Payoff = max{0, X – S_T}

- S_T = price of the underlying asset at the maturity date (T)
- X = exercise price

Replication Model of Valuing:

- $Price_U$ S + (1+r)B = $Payoff_U$
- $Price_D$ S + (1+r)B = $Payoff_D$

Binomial Model of Valuing:

- Value at any time: $V_{t-1} = E(V_t) / (1+r) = (qV_U + (1 – q)V_D) / (1 + r)$
- Risk-Adjusted Probability: $q = ((1+r)S_0 – S_D)/(S_U – S_D)$
- If only volatility is known:
  - $u = e^{\sigma\sqrt{T}}$
  - $d = 1/u$
  - $q = (1 + r – d)/(u – d)$
  - $S_U = uS_0$, $S_D = dS_0$

Put-Call Parity: $P_t = C_t – S_t + X/(1+r)^{T-t}$

- If $S_t = X$ and T is small enough, $P_t = C_t$.