Consider a linear potential of the form

$$V(r) = \alpha r - V_0.$$ 

As usual, the Schrödinger equation can be rewritten in terms of the $\hat{L}^2$ operator, and since this potential is still central, the wavefunction will separate into the product of a radial equation and a spherical harmonic. Applying SOV, the Schrödinger equation will look like

$$-\frac{\hbar^2}{2m} \left( \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dR}{dr} \right) - \frac{\ell(\ell + 1)}{r^2} R \right) + (\alpha r - V_0 - E) R = 0.$$ 

Substituting $\chi(r) = rR(r)$, this equation becomes

$$-\frac{\hbar^2}{2m} \frac{d^2\chi}{dr^2} = (\alpha r - V_0 - E + \frac{\hbar^2\ell(\ell + 1)}{2mr^2}) \chi = 0.$$ 

For $\ell > 0$, we must use numerical methods to solve this equation. For $\ell = 0$, it simplifies to

$$-\frac{\hbar^2}{2m} \frac{d^2\chi}{dr^2} = (\alpha r - V_0 - E) \chi = 0.$$ 

Making the substitution

$$\xi = \left( r - \frac{E + V_0}{\alpha} \right) \left( \frac{2ma}{\hbar^2} \right)^{1/3}$$ 

and doing some algebra, we have

$$\frac{d^2\chi}{d\xi^2} = \xi \chi,$$

which is Airy’s equation. The solutions are

$$\chi(\xi) = CAi(\xi),$$

where we have excluded the non-normalizable $Bi(\xi)$ solution. The energy eigenvalues are given implicitly by setting $r = 0$, which amounts to the condition that

$$Ai \left( \frac{-E + V_0}{\alpha} \left( \frac{2ma}{\hbar^2} \right)^{1/3} \right) = 0.$$ 

Thus, the energies depend on the zeros of the Airy functions and are given by

$$E_n = -z_n \left( \frac{\hbar^2 \alpha^2}{2m} \right)^{1/3} - V_0,$$

where $z_n$ is the $n^{th}$ zero of $Ai(\xi)$. 