Dispersion in Prisms

Cauchy’s equation (which is a special case of Sellmeier’s equation, as I showed two weeks ago) gives \( n(\lambda) \) when considering wavelengths away from resonance regions:

\[
n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots.
\]  

These constants are generally determined by fitting empirical data. The dispersion of a material is defined as \( \frac{dn}{d\lambda} \), which is approximately

\[
\frac{dn}{d\lambda} = -\frac{2B}{\lambda^3} - \frac{4C}{\lambda^5}.
\]  

Note that dispersion is different from the deviation for a prism described in Hecht. Large dispersion means a large angle between where the light would have been if the prism were not in its path and where it ends up after passing through the prism. Large deviation means a large angle between light of different wavelengths after passing through the prism.

The dispersive power of a prism is the ratio of the dispersion to the deviation:

\[
\Delta = \frac{D}{\delta} = \frac{n_B - n_R}{n_Y - 1},
\]  

where \( n_B \) is the index of refraction for 492-455 nm blue light, \( n_Y \) is for 597-577 nm yellow light, and \( n_R \) is for 780-622 nm red light.

As the wavelength difference between components of light incident on a prism decreases, the ability of the prism to resolve them begins to fail. Using the pictured geometry and Rayleigh’s criterion, which gives the minimum resolvable separation between two wavefronts as \( \Delta \alpha = \lambda/d \), we find a resolving power of

\[
\mathcal{R} = \frac{\lambda}{(\Delta \lambda)_{\text{min}}} = b \frac{dn}{d\lambda},
\]  

where \( (\Delta \lambda)_{\text{min}} \) is the minimum wavelength separation permissible for resolvable images.