Low Temperature Resistivity

Lisa M. Larrimore

The temperature dependence of electrical resistivity was measured for three materials: a conductor (copper), a semiconductor (a carbon resistor), and a doped semiconductor (a pn junction). As expected, the resistance of the copper sample decreased with temperature and the resistance of the carbon resistor increased with temperature. The temperature coefficient of resistance of Cu was calculated to be $\alpha = 0.0040 \pm 0.0002$ 1/C; the accepted value of 0.0039 is within this uncertainty. From the nonlinear behavior of the pn junction, the the band gap energy of pure Si was calculated to be $E_G = 1.2 \pm 0.1$ eV; the accepted value of 1.12 eV is within this uncertainty. No quantitative measurements could be made from the carbon data, due to limitations in the theory and to the fact that a carbon resistor is not a pure semiconductor.

Introduction

Measuring the temperature dependence of resistivity reveals some important differences between metals, insulators, and semiconductors. There are a variety of factors that influence resistivity (and thus its inverse, conductivity), and identifying these mechanisms and fitting the experimental data can reveal properties of the material, such as the band gap energy, E_G, or the temperature coefficient of resistance, α.

There are two important factors that contribute to the conductivity of a material: the availability of free electrons and the ability of these electrons to move freely through a material. For a conducting metal with free valence electrons, the factor limiting conductivity is the lattice vibrations which scatter moving electrons; as temperature decreases, these vibrations (and thus the resistivity) also decrease. In this investigation, a copper sample is used to demonstrate the resistivity of a metal.

In an insulator, the electrons are unable to break free from the filled valence band, so there is little conduction. For a semiconductor, the energy gap is small enough that there

are some free electrons at higher temperatures. Since the factor limiting conductivity for a semiconductor is the availability of free electrons, resistance increases with decreasing temperature, which is the reverse of the behavior observed in a metal [1]. A pure semiconductor is a perfect crystal; since such chemical purity does not naturally occur, synthesized crystals have been used to test theoretical calculations [2]. We used a carbon resistor to simulate the resistivity of a semiconductor.

Adding small impurities to a pure intrinsic semiconductor results in an extrinsic doped semiconductor, which has a nonlinear relationship between current and voltage. We used a pn junction to study the behavior of doped semiconductors. Interestingly, the added impurities to our silicon sample could be used to determine an important property of pure Si, the band gap energy, E_G.

Theory

In a conducting metal, there are always plenty of charge carriers to carry a current, so the factor that determines resistivity, ρ, is their ability to move freely through the material. Atoms in a solid are confined to a site in the lattice structure, and at 0 K, the motion of an electron through the lattice is unimpeded. At higher temperatures, however, the atoms vibrate around their equilibrium positions, resulting in scattering of free electrons moving through the solid [1]. These vibrations can be modeled in various ways; in the Einstein model, for example, each atom is considered as an independent oscillator. A more realistic model is the Debye model, in which each atom undergoes coupled oscillations with its neighboring atoms [3].

The quantum of energy of a lattice vibration is a phonon, and just as there are only certain photon modes allowed in a laser cavity, only certain phonon modes are allowed in a crystal. The average number of phonons in a given mode is described by the Planck distribution,

$$\langle n \rangle = \frac{1}{e^{\hbar\omega/kT} - 1}. \tag{1}$$

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In the Debye model, the density of these modes is given by

$$D(\omega) = \frac{V\omega^2}{2\pi^2 c_s^3},$$

(2)

where V is the volume of the solid and c_s is the speed of sound [4]. The total average number of phonons in the lattice is given by

$$\langle n_{\text{total}} \rangle = \int D(\omega) \langle n \rangle \, d\omega.$$

(3)

For a unit volume and counting for the three phonon polarizations, this results in

$$\langle n_{\text{total}} \rangle = \frac{3}{2\pi^2 c_s^3} \int_0^{\omega_D} \frac{\omega^2}{e^{\hbar\omega/kT} - 1} d\omega,$$

(4)

where ω_D is the maximum mode that can be supported by the solid. The Debye temperature is related to the maximum mode frequency by $\theta_D = \hbar\omega_D/k$. For $T \ll \theta_D$, $\langle n_{\text{total}} \rangle$ goes as T^3, and for $T \gg \theta_D$, $\langle n_{\text{total}} \rangle$ goes linearly with T [5]. Since more oscillations make it more difficult for an electron to move through a solid, we make resistivity proportional to $\langle n_{\text{total}} \rangle$.

In the linear regime, the resistance of a conductor is given by

$$R = R_0 \left[1 + \alpha(T - T_0) \right],$$

(5)

where α is the temperature coefficient of resistance measured at temperature T_0. At $T_0 = 20C = 293K$, the accepted value of the temperature coefficient for Cu is $\alpha = 0.0039$ 1/C [6].

For a pure semiconductor, phonon vibrations still impede the motion of charge carriers at higher temperatures, but the availability of these charge carriers is the most important factor in determining resistivity. At absolute zero, the valence band in a semiconductor is completely full, and the conduction band is empty. The energy that an electron needs to jump to the conduction band is known as the band gap energy, E_G, which is typically between 0.5 eV and 2 eV for a semiconductor. The relationship between resistivity and temperature for a pure semiconductor is

$$\rho \propto T^{-3/2} e^{E_G/kT},$$

(6)

where the exponential behavior dominates at low temperatures [5].

For a conductor and a pure semiconductor, the current increases linearly with voltage, so the resistance can be determined using Ohm's law, $V = IR$. For a pn junction, however, there is a preferential direction for current flow, and the relationship between voltage and current is nonlinear. A pn junction is the connection of an n-type semiconductor, in which there are excess electrons, with a p-type semiconductor, in which there are missing electrons. The places where electrons are missing are known as holes, and they behave as positive charge carriers. In this experiment, the pn junction is made from silicon with added dopant. The electrons in the n-type semiconductor and the holes in the p-type semiconductor are the majority carriers, and when a positive voltage is applied from the p region to the n region, there is an exponential increase in current, $e^{qV/kT}$. When a negative voltage is applied, there is no enhancement due to the majority charge carriers. For any applied voltage, there is also a small current carried by the minority carriers, which are the electrons or holes from the silicon atoms that would be the charge carriers in the conduction band for undoped silicon. These effects are expressed in a simple model as

$$I = I_0 \left(e^{qV/kT} - 1 \right), \tag{7}$$

where q is the charge of an electron, V is the voltage across the junction, and I_0 is proportional to the diffusion current density, or the current carried by the minority carriers. Just as for a pure semiconductor the number of charge carriers has a $e^{-E_G/kT}$ behavior, we expect I_0 to have the same proportionality. The energy in the exponent, however, is really $E_G - E_F$, where E_F is the Fermi energy. For intrinsic semiconductors, the Fermi energy generally falls midway between the valence and conduction bands [2]. We can thus write I_0 as

$$I_0(T) \propto e^{-E_G/2kT}, \tag{8}$$

where E_G is the band gap of pure silicon, which has an accepted value of 1.12 eV [7].

Procedure

Each sample was prepared using a four-point probe, which minimized the error due to simultaneously measuring voltage and current by preventing much current from flowing

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through the voltmeter connected across the sample. This voltmeter was connected across the inner two electrodes, and a current ran through the outer two. In the outer circuit was a power supply and a 10k precision resistor. By measuring the voltage across the precision resistor, we could use Ohm's law to determine the current running through the sample for the metal and the semiconductor. The resistance of the sample was calculated by dividing the voltage drop across it by this current. For the doped semiconductor, the relationship between voltage and current was nonlinear, and a simple model was used to describe this relationship.

Each was sealed in a vacuum, which was achieved using an oil pump and a liquid nitrogen (LN_2) trap. The samples could then be cooled to nearly 10 K using a liquid helium refrigerator. Starting at 300 K, the resistances of the copper and carbon samples were measured in 25 K increments down to 50 K, and then in 10 K increments to as low a temperature as possible. The resistance of the pn junction was measured in 25 K increments between 300 K and 150 K.

Results and Discussion

Figure (1) shows the resistance of copper as a function of absolute temperature for low temperatures. In this region, the resistance is cubic with temperature, as the Debye theory suggests. Cu resistance at higher temperatures is shown in Figure (2). Though the accepted Debye temperature of Copper is 343 K [4], which is higher than any temperature we measured, our data appears linear until the temperature falls below 75 K. While the Debye model gives the correct limiting behavior for the resistance (cubic at low temperatures, linear at higher temperatures), it does not accurately predict where this transition occurs. By using the curve fit parameters from Figure (2) in Equation (5), and using $T_0 = 20$ C, we obtain a temperature coefficient of $\alpha = 0.0040 \pm 0.0002$ 1/C. The accepted value of 0.0039 1/C is within this uncertainty.

Figure (3) shows the resistance of the carbon resistor as a function of temperature. As expected, the resistance increases as the temperature decreases and fewer charge carriers

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become available. Trying to fit the data to the quantitative model given by Eq. (6) was unsuccessful. There is a region, shown in Figure (4), where $\ln R$ goes linearly with $1/T$, but the slope of this line is not E_G/k: it gives $E_G = 0.001$ eV, which is far too small. Some disagreement with theory is to be expected, since carbon is only used to approximate the behavior of a pure semiconductor. It seems that we can only make the qualitative conclusion that resistance goes up with decreasing temperature.

Figure (5) illustrates the current through the pn junction as a function of temperature at 300 K. According to Eq. (7), the relationship should be $I = I_0 \left(e^{qV/kT} - 1 \right)$, but there was no I_0 for which this accurately described our data. This is illustrated in the lower curve fit in Figure (5). Allowing q/kT to be a parameter, rather than using its accepted value, resulted in a much better fit, as shown.

At low temperatures, the currents measured through a pn junction become very small, and the model given in Eq. (7) becomes more flawed as other effects become significant. We therefore chose to only consider the data taken above 200 K when calculating the band gap energy. These data are shown in Figure (6), fit using Eq. (8). This results in a band gap energy of 1.2 ± 0.1 eV. The accepted value of E_G for Si of 1.12 eV is within the uncertainty of our measurement.

Conclusion

The temperature dependence of resistance was measured for three materials, illustrating the two mechanisms contributing to resistance: availability of charge carriers and ability of charge carriers to move unimpeded through the solid. The resistivity of the copper sample was controlled by the latter effect, which caused its resistance to decrease as we decreased the temperature and thus the number of phonon vibrations. The resistivity of the carbon resistor was controlled by the former effect, which caused its resistance to increase as we decreased the temperature and thus the number of available charge carriers.

Studying the resistivity of these materials also allowed us to calculate some of their properties. We calculated the temperature coefficient for Cu to be $\alpha = 0.0040 \pm 0.0002$ 1/C,

which includes the accepted value of 0.0039 1/C in its uncertainty. We also found the band gap energy of pure Si, $E_G = 1.2 \pm 0.1$ eV, and this uncertainty range contains the accepted value of 1.12 eV. No quantitative measurements could be made from the carbon data, due to limitations in the theory and to the fact that a carbon resistor is not a pure semiconductor.
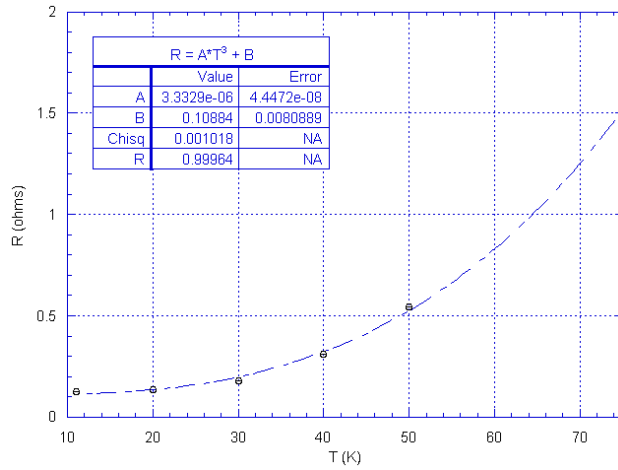
Figure 1: Resistance of a copper sample at low temperatures. As Debye theory suggests, $\rho \propto T^3$. The error bars are smaller than each circle.
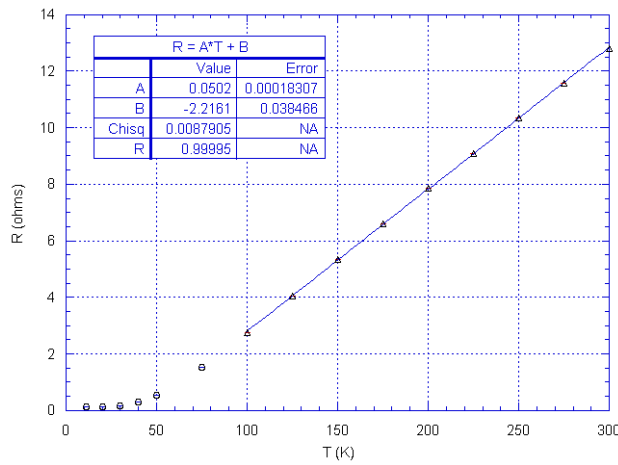

Figure 2: Resistance of a copper sample as a function of temperature. Open circles are data from Figure (1), triangles are data that did not fit to cubic function. The slope of the linear fit can be used to calculate the temperature coefficient of resistivity.
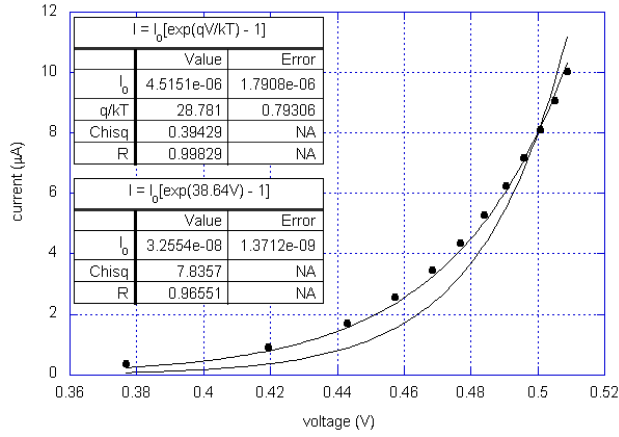
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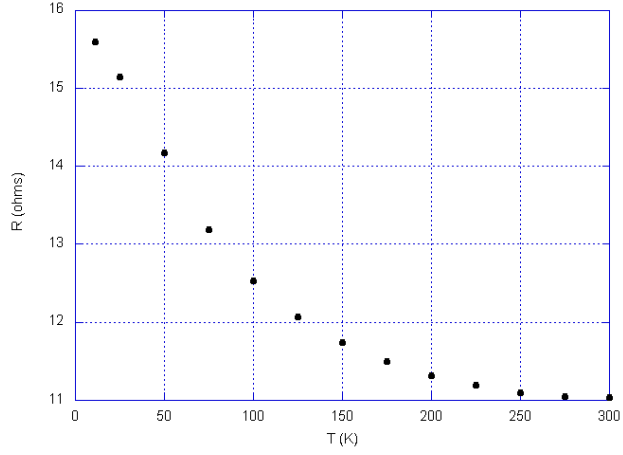
Figure 3: Temperature dependence of resistance of carbon resistor. Attempts to fit this to the curve given in Eq. (6) were unsuccessful.
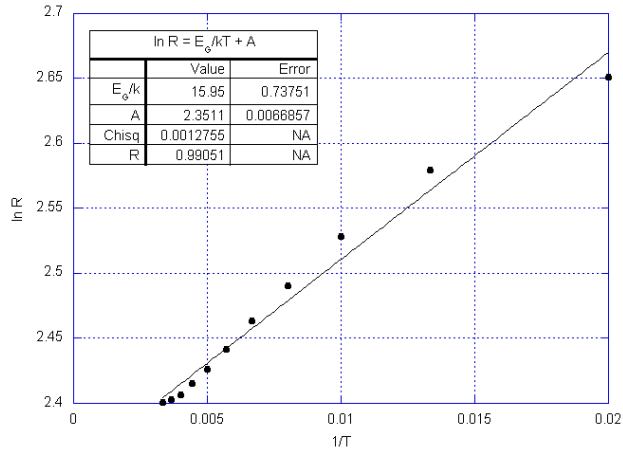
Figure 4: $\ln R$ plotted against $1/T$ for carbon resistor. If $R \propto e^{E_G/kT}$, the slope of this line should be E_G/k. However, this gives $E_G = 0.001$ eV, which is far too small.

Figure 5: Current through a pn junction as a function of voltage at 300 K. The curve fit that does not go through the data, in which I_0 is the only parameter, illustrates that the simple model in Eq. (7) is not the whole story. To more accurately find I_0, we let q/kT be a parameter of the curve fit as well.
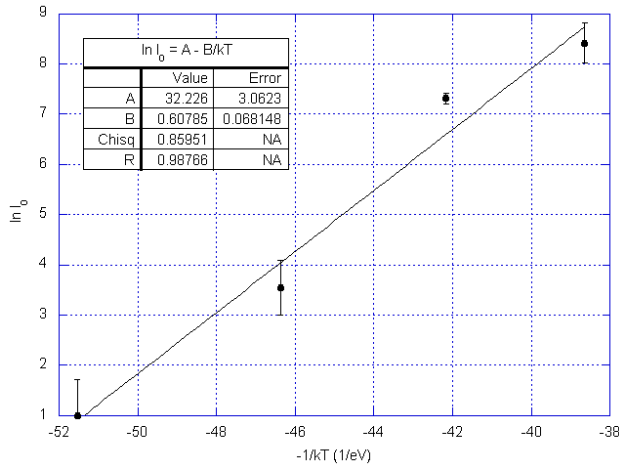
Figure 6: The value of I_0 at different temperatures can be used to calculate the energy gap E_G of pure silicon, as shown in Eq. (8). The curve fit shown here results in $E_G = 1.2 \pm 0.1$ eV.

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