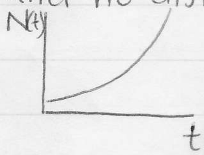
Population Growth Models

<u>Importance</u>: predict populations, effects on economy, natural resources, the environment

- Two main simple models, logistic and exponential, differ in assumptions

- <u>Exponential</u>: THOMAS MALTHUS
  - unlimited resources, no interaction with other species, constant birth and death rates that are proportional to the population, closed population, and no distinction (age, reproduction)



t = time

$N(t)$ = population with respect to time

$$\frac{dN}{dt} = (b-d)N(t) = rN(t)$$

$$\frac{dN}{dt} = rN(t) \to \frac{dN}{N(t)} = r\,dt \to \int \frac{dN}{N(t)} = \int r\,dt \to \ln|N(t)| = rt + c$$

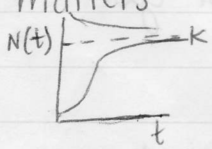
$$N(t) = e^{rt+c} = Ae^{rt} \quad \text{Assume } N(0) = N_0 \text{, so } A = N_0$$

$$\boxed{N(t) = N_0 e^{rt}}$$

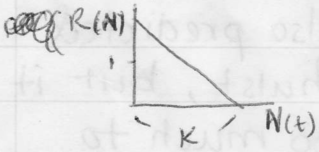
- Useful for modeling bacteria growth and early population growth, when conditions are favorable.

- <u>Logistic</u>: Pierre Verhulst
  - assumes the same things as exponential model, but instead of unlimited resources, resources are limited and population density matters

S-shaped curve, called Sigmoid



K = carrying capacity, equilibrium → each individual replaces itself. When the population size is near 0, the rate of increase (intrinsic rate of growth), measures when no competition for resources (r_d)



$R(N)$ starts at $1 + r_d$, so in slope-intercept form

$$R(N) = (1 + r_d) + \left(-\frac{r_d}{K}\right)N(t) \quad \text{multiply and stuff}$$

$$\frac{dN}{dt} = r_d N(t)\left(1 - \frac{N(t)}{K}\right)$$

$N = N(t)$ for simplicity

Solve