$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) \qquad \frac{dN}{N(1-\frac{N}{K})} = r\,dt \qquad \int \frac{dN}{N(1-\frac{N}{K})} = \int r\,dt$$

Partial Fractions $\quad \frac{1}{N(1-\frac{N}{K})} = \frac{A}{N} + \frac{B}{1-\frac{N}{K}} \qquad 1 = A(1-\frac{N}{K}) + BN$

If $N=0$, $\quad 1 = A(1-\frac{0}{K}) + B(0) = A(1)$, $\quad A=1$

If $N=K$, $\quad 1 = A(1-\frac{K}{K}) + BK = BK$, $\quad B = 1/K$ $\quad$ Plug back in

$$\frac{1}{N(1-\frac{N}{K})} = \frac{1}{N} + \frac{1/K}{1-N/K} \rightarrow \int\left(\frac{1}{N} + \frac{1/K}{1-N/K}\right)dN = \int r\,dt$$

$$\ln|N| - \ln|1-N/K| = \ln\left|\frac{N}{1-N/K}\right| = rt + C$$

$$\frac{N}{1-N/K} = e^{rt+c} = Ae^{rt} \qquad \frac{N}{1-\frac{N}{K}} = \frac{N}{\frac{K}{K}-\frac{N}{K}} = \frac{N}{\frac{K-N}{K}} = \frac{KN}{K-N}$$

$$\frac{KN}{K-N} = Ae^{rt} \rightarrow N = \frac{KAe^{rt} - NAe^{rt}}{K} = Ae^{rt} - \frac{NAe^{rt}}{K}$$

$$N + \frac{NAe^{rt}}{K} = Ae^{rt}$$

$$\frac{NK + NAe^{rt}}{K} = Ae^{rt} = \frac{N(K+Ae^{rt})}{K} \rightarrow N = \frac{KAe^{rt}}{K+Ae^{rt}}$$

If $N(0) = N_0$ $\quad N_0 = \frac{KAe^{r(0)}}{K+Ae^{r(0)}} = \frac{KA}{K+A} \qquad \frac{K+A}{KA} = \frac{1}{N_0} = \frac{K}{KA} + \frac{A}{KA} = \frac{1}{A} + \frac{1}{K}$

$\frac{1}{N_0} - \frac{1}{K} = \frac{K}{KN_0} - \frac{N_0}{KN_0} = \frac{K-N_0}{KN_0} = \frac{1}{A} \qquad A = \frac{KN_0}{K-N_0}$ $\quad$ Plug back in

With a lot of manipulation, you get

$$N(t) = \frac{KN_0}{N_0 + (K-N_0)e^{-rt}}$$

As $t \to \infty$, $(K-N_0)e^{-rt} \to 0$, so $N(t) \to K$, the carrying capacity.

- Useful for modeling many things, like yeast. Also predicted human population somewhat accurately for Verhulst, but it is not a very good model for humans; too much to take into account.